The Weighted Majority Rule With Less Than Fully Competent Voters

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1 Introduction

The weighted majority rule is a somewhat unorthodox democratic decision method, proposed in two separate papers by Brighouse and Fleurbaey (2010) and Fleurbaey (mimeo). In the standard case, the rule is applied to decisions with two options, i.e. *binary* decisions. The weighted majority rule states that every person's vote is to be assigned a voting weight in proportion to what is at stake for this person in the decision, and that the option that receives more voting weights is selected as the outcome, or winner, of the collective decision.

> ***The Weighted Majority Rule:*** For all individuals and any decision with two options, (a) every individual is assigned a number of votes in proportion to her stakes, and (b) the option that receives a majority of votes is selected as outcome.

As stated by Brighouse and Fleurbaey, the weighted majority rule assigns varying voting weights to votes. I find it easier to present my examples and arguments in terms of varying numbers of votes, so this is what I do in the remainder of this study. But this way of speaking should not suggest that voters could split their votes between different options. Rather, when I speak of numbers of votes assigned to some voter, this should be understood as an indivisible vote bundle.

Fleurbaey (mimeo) considers the collective optimality, in terms of the sum total of utility, of the weighted majority rule, given the following conditions:

> ***Welfare Stakes:*** An individual's stakes in a binary decision consist in her differential in well-being between the two options.

> ***Self-Interested Voting:*** Every voter (that is, individual who has been assigned a positive number of votes) votes according to her self-interest.

Fleurbaey then provides a proof for what I will call the Original Theorem:

> ***The Original Theorem:*** For all individuals and any decision with two options, given the Weighted Majority Rule, Welfare Stakes, and Self-Interested Voting, the option with the greater sum-total of well-being is selected as the outcome.[1]

The assumption of Self-Interested Voting is a rather strong one. However, it can be easily seen that the assumption can be relaxed (rendered logically weaker), while retaining the collective optimality of the weighted majority rule.[2] Consider a binary issue and a group of exclusively self-interested voters, for which the Original Theorem has been shown to hold. Now let one of these voters turn into a common-

[1] Brighouse and Fleurbaey (2010) moreover provide an argument for the collective optimality of the weighted majority rule in prioritarian terms; cf. Berndt Rasmussen (2013: 56-58). I have shown elsewhere that the argument for the collective optimality of the weighted majority rule can be adapted to a wider variety of criteria of the common good, including sufficientarian, maximin, leximin, and non-welfarist, e.g. autonomy-constrained criteria (Berndt Rasmussen 2013: 58-63).

[2] Cf. Berndt Rasmussen 2013: 71-73.

interested voter instead. Then either this does not change her vote (since her self-interest happened to coincide with the common interest), and thus does not change the outcome in any way. Or this does change her vote (since her self-interest was opposed to the common interest); then, the common interest option will merely get an "extra" vote, and thus still be chosen by the weighted majority rule. Hence, the outcome does not change either way. The same argument can be made for any additional voter who turns from self-interested to common-interested. Thus, the Original Theorem must hold for all cases of exclusively common-interested voters, as well as "mixed" cases.

That is:

> ***Self- or Common-Interested Voting:*** Every voter (that is, individual who has been assigned a positive number of votes) votes according to her self-interest or according to the common interest.

> ***The Extended Theorem:*** For all individuals and any decision with two options, given the Weighted Majority Rule, Welfare Stakes, and Self- or Common-Interested Voting, the option with the greater sum-total of well-being is selected as the outcome.

Note, however, that the theorem cannot be extended further to cover even voters who vote according to any "partial" interest that conflicts with their self- or the common interest.[3] The condition of Self- or Common-Interested Voting is, however, still quite strong. Given a folk psychological picture of reasons for action, we might interpret this condition as satisfied if voters are motivated by their self- or the common interest and fully competent in assessing the available options in light of these criteria.

> ***Self- or Common-Interested Motivation:*** Every voter is either self-interested, in the sense that she desires to promote her self-interest, or common-interested, in the sense that she desires to promote the common interest.

> ***Competence:*** Every voter has a correct belief that voting for some option *x*, among the given alternatives, promotes her self-interest (if she is self-interested) or the common-interest (if she is common-interested).

It seems that the argument for the weighted majority rule would be considerably strengthened if it could be shown that these assumptions can be relaxed. In this paper, I will set out to explore how far Competence may be relaxed, while preserving the collective optimality of the weighted majority rule. In the course of my arguments, it will be shown that there is some room to also relax the Self- or Common-Interested Motivation assumption.

My arguments build on a number of theorems that are generalisations and extensions of the well-known Condorcet jury theorem. I argue that these Condorcet theorems can be applied to the weighted majority rule under conditions of (empirical) uncertainty to show that this rule selects the common-interest option with *near certainty*, even when Competence is not satisfied. The upshot is that the weighted majority rule can still be shown to be collectively optimal, in a weaker sense. I define 'being weakly collectively optimal' as 'selecting the collectively optimal option with near certainty or certainty'. The arguments in this paper show that the weighted majority rule is *weakly* collectively optimal. This conclusion is, obviously, weaker than the ones provided by the Original and Extended theorems. It rests, however, on an improved argument that does not rely on the strong assumptions of Competence and — as a corollary — of Self- or Common-Interested Motivation.

[3] Cf. Berndt Rasmussen 2013: 73-74

I introduce the classical Condorcet jury theorem in 2.1. I then proceed to show how various generalisations and extensions of it are relevant to the present study. First, I consider only cases where everyone has equal stakes. I show that some of the Condorcet theorems apply to them, by arguing that they apply to cases where all voters are common-interested (2.2) and that this also holds when all voters are assumed to be self-interested (2.3). I then proceed to show that my arguments can be extended to 'mixed' cases with both self- and common-interested voters (2.4). In 2.5, I relax this assumption further by allowing more heterogeneous patterns of voter competence. I proceed to show how my analysis helps us solve the so-called Mixed motivation problem. Second, I consider cases where stakes are unequal and where voters are thus assigned unequally sized bundles of votes. In 2.6, I show that a crucial assumption for Condorcet theorems is violated in these cases, namely that of Independence. I then show that the dependence resulting from vote bundles can be dealt with through yet another Condorcet theorem (2.7). Finally, in section 3, I sum up and conclude.[4]

2 The argument from weak collective optimality

Let's imagine a group of individuals who face a binary decision and whose stakes are defined in accordance with Welfare Stakes. More specifically, all these stake-holders ('voters') happen to be common-interested, thereby complying with Self- or Common-Interested Motivation. However, they do not correctly judge which option is in the common interest and thus fail to satisfy Competence. Then, Self- or Common-Interested Voting does not follow, and thus the Extended Theorem, according to which the weighted majority rule is collectively optimal, does not hold.

There is an influential theorem regarding the results of *simple* majority rule with less than fully competent voters: the Condorcet jury theorem. I now state it and then apply it — in a series of steps — to the weighted majority rule.

To clarify some terminological issues: by 'voter competence' or 'individual competence' (c_i) I refer to the probability that voter i correctly judges the options (according to a specified standard). If this probability is one ($c_i = 1$), I call the voter 'fully competent'. If it is greater than chance ($c_i > ½$), I call her 'minimally competent'. If it is zero ($c_i = 0$), I call her 'incompetent'. By 'group competence' (P_n) I refer to the probability that the majority of the group votes for the correct judgment.[5]

[4] I will utilise Condorcet theorems, that have been developed and proved by other theorists, for my purposes. My main strategy is to show that the theorems' assumptions hold in the cases I discuss and hence that their results apply here as well. It is, however, not the task of the present study to analyse or question the theorems and proofs themselves. This means that my arguments are sound only to the extent that these theorems and proofs are correct.

[5] I do not presuppose a specific interpretation of 'probability'. Note, though, that a subjectivist interpretation, identifying each voter's probability of a correct judgment with the degree of confidence that she herself assigns to her judgment, is not suitable here. However, a number of objectivist interpretations could be adopted e.g. a frequency interpretation (identifying the voter's probability of judging correctly with her relative frequency of correct judgments in a series of similar events) or a propensity interpretation (identifying this probability with the voter's tendency or disposition to judge correctly). What is important, though, is that the same interpretation is used consistently throughout the text.

2.1 Introducing the Condorcet jury theorem

In its classical version, the Condorcet jury theorem applies to binary decisions between two propositions. Such a pair of propositions can be expressed, e.g. by the claim 'The defendant is guilty' and its negation. It is assumed that only one of the claims in each pair is correct according to an independent standard (e.g. a standard of being guilty or of being in the common interest). Moreover, the following definitions are presupposed.

> ***Equal Minimal Competence:*** Every voter is equally likely to judge the options correctly and somewhat more likely than not to judge the options correctly, such that every voter's individual competence is $c > ½$.
>
> ***Voting According to Judgment:*** Every voter votes according to this judgment.
>
> ***Voter Independence:*** Every voter judges (probabilistically) independently, that is, how each judges the options does not depend on how others judge them.

The Condorcet jury theorem (CJT), in its classical form, then states:

> ***Classical CJT:*** For binary decisions with exactly one correct option (according to some independent standard) and a group of $n = 2m+1$ voters, given *Equal Minimal Competence*, *Voting According to Judgment*, and *Voter Independence*, (i) the probability of a correct majority vote P_n is higher than any single voter's competence c, and (ii) P_n strictly increases with the number of voters n and approaches certainty as the number of voters increases to infinity.[6]

The theorem takes the form of a conditional, stating three conditions and a derived result. Concerning this result, claim (i) states that it is more likely that the majority votes for the correct option than that any single voter does. This is sometimes labelled the *non-asymptotic conclusion*. Claim (ii) states that this gets more and more likely (approaching certainty) as the group size increases (toward an infinite number of voters). This is sometimes labelled the *asymptotic conclusion*. Since my arguments below do not depend on the non-asymptotic conclusion, I will omit it in the remainder of this paper. The asymptotic conclusion will be somewhat modified by the below extensions of the Condorcet jury theorem. For convenience, I keep the label even for these slightly different claims.

How is the Condorcet jury theorem supposed to apply to my present study? This is by no means obvious. Consider that Classical CJT presupposes, first, that there is one correct option, according to an *independent standard of correctness*. Second, it presupposes that all voters vote for what they judge

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[6] For the original results, see Condorcet (1785). Cf. e.g. Ladha (1992: 618), Miller (1986: 175) and Boland (1989: 182). Owen, Grofman and Feld (1989: 2) also make explicit the assumption that the 'prior odds as to which of the two alternatives is the correct one are even'. For a short proof of Classical CJT, see e.g. Ladha (1992: 632f.). In fact, Ladha's proof is conducted for any (even or odd) number $n \geq 3$ of voters. Since some of the subsequent theorems are formulated for odd-numbered groups, I stick to this assumption from the outset for the sake of simplicity. The limitation to odd-numbered groups is common in the literature because it simplifies the exposition considerably. Still, it should be noted that the theorem can be extended to even-numbered cases of more than three voters. To be precise: for claim (ii), with an even-numbered group of $n > 3$ voters, if ties are resolved by a random tie-breaker, the probability that a majority of these n voters votes for the correct option equals the probability that a majority of $n–1$ voters votes for the correct option (cf. e.g. Miller 1986: 175; Ladha 1992: 618). That is, the probability of a correct majority vote is equivalent to that of the next-smaller odd-numbered group of voters. For claim (i), with an even-numbered group of $n > 3$ voters, the probability of a correct majority vote is greater than any single voter's competence conditional on a stricter competence assumption. E.g. for $n = 4$ voters, we need to assume $c > 0.77$; for $n = 12$, $c > 0.56$; and for $n = 102$, $c > 0.5056$ (Bovens and Rabinowicz 2006: 4f.; 36). That is, voters must be significantly better than chance, but with a margin that decreases as the number of voters increases.

to be the correct option, so implicitly, we take voters to be motivational truth- or *correctness-trackers*. Third, it presupposes that the decision is made by *simple* majority rule.

As a first objection against Classical CJT's applicability, one might suggest that there is no correct option here, according to some independent standard of correctness. However, this is clearly not true. We have assumed a sum-total criterion, which gives us the independent standard of 'being the option with at least as high sum-total of well-being as any other'.[7] If we disregard cases where both options are equally good (since it does not matter, ceteris paribus, which of them is selected), this independent standard singles out, for any binary decision, exactly one option as the common-interest option. We may then say that this is the correct option, according to the independent standard of the accepted criterion.

This standard is of course not entirely independent of the voters, as the sum-total of well-being is constituted by the aggregate of the voters' well-being. But it is independent in the relevant sense since it does not causally depend on their votes and underlying judgments.[8]

As a second objection, one may claim that Classical CJT can only be applied to very specific cases: those where all voters judge the options in the light of the proposed independent standard — that is, those cases where all voters are common-interested. Yet we want to say something about cases with self-interested voters (or mixed cases) as well. The problem with self-interested voters is that they vote not according to their judgment of an independent standard of the common interest, which singles out one and the same option for all voters. Rather they vote according to their perception of their respective individual standard of self-interest, which may pick out one option as better for some of them, and the other option as better for others.

This is indeed a considerable complication. I deal with it in the following three sections. My argument, to the effect that Classical CJT is applicable, focuses first on groups of exclusively common-interested voters, is in a second step extended to hold for self-interested voters and in a third step for mixed groups of both kinds of voters.

As a third objection, one may note that Classical CJT is meant to apply to *simple* majority rule, while the present study deals with the *weighted* majority rule. Again, this is a complication I deal with below. Simple majority rule is a special instance of the weighted majority rule, for binary cases where everyone's stakes are equal. So I start out by only considering equal-stakes cases. For these cases, a number of Condorcet theorems can be directly applied to the weighted majority rule. I eventually extend the results to unequal-stakes cases, where simple and weighted majority rule differ (in their assignments of votes and hence, possibly, in their outcomes).

A fourth objection is that applying Classical CJT is not much of an improvement: though it allows us to relax the strong *Competence* assumption, it brings with it pretty strong assumptions of its own, namely competence equality (*Equal Minimal Competence*) and an additional assumption (*Independence*). In the course of this paper, I explore how these new assumptions can be relaxed as well.

[7] My arguments below can be adapted to other criteria (see footnote 3 above; cf. Berndt Rasmussen 2013). It is not important *which* specific criterion we have accepted, only *that* we have accepted one.

[8] Note that the equal minimal competence assumption implies that these judgments are truth- or correctness-tracking, to some degree. Thus, there is probabilistic dependence between the independent standard and the voters' judgments. I return to different forms of dependence and their relevance for my arguments in 2.6 below.

2.2 Equal-stakes cases with common-interested voters

Let us start then by considering cases where all voters have equal stakes and all are common-interested: their desired end is the promotion of the common interest. They are not motivational correctness-trackers whose desired end is to find out the correct option. This is, however, not an obstacle for applying Classical CJT. The precise nature of the voters' ends does not matter. What matters is that they, as common-interested voters, vote according to their judgment of the correct option, according to the independent standard of the common good (as assumed in Voting According to Judgment).

Hence, for these specific cases, Classical CJT can be employed as the first premise in an argument for the weighted majority rule. As we will see shortly, the argument's conclusion (6) is simply an adaptation of Classical CJT to the present setting of equal-stakes cases with common-interested voters. This conclusion then specifies the conditions under which the weighted majority rule is weakly collectively optimal (that is, selects the common-interest option with near certainty or certainty) in these cases. Let us then call what follows the *first argument from weak collective optimality*.

> **(1)** *Classical CJT.* For binary decisions with exactly one correct option (according to some independent standard) and a group of n = 2m+1 voters, given Equal Minimal Competence, Voting According to Judgment, and Voter independence, the probability of a correct majority vote P_n strictly increases with the number of voters n and approaches certainty as the number of voters increases to infinity.
>
> **(2)** The correct option is the common-interest option, according to the independent standard of the given criterion of the common good.
>
> **(3)** For equal-stakes binary decisions, the option for which a majority of voters votes is selected by the weighted majority rule (since the rule assigns equal votes to all voters and selects the option that receives a majority of votes).
>
> **(4)** For any given level of equal minimal voter competence, there is a range of probability levels of a correct majority vote that we call 'near certainty' and (according to the Condorcet jury theorem) a corresponding range of numbers of voters n that we call 'sufficiently large numbers' of voters.
>
> **(5)** A rule that with near certainty selects the common-interest option, as defined by the given criterion of the common good, is weakly collectively optimal, according to this criterion.
>
> Hence:
>
> **(6)** For equal-stakes binary decisions with exactly one common-interest option (according to the given criterion of the common good) and a group of a sufficiently large number n = 2m+1 of common-interested voters, given Equal Minimal Competence, Voting According to Judgment, and Voter Independence, the weighted majority rule is weakly collectively optimal (according to this criterion).

Thus, for equal-stakes cases with common-interested voters, the weighted majority rule can be shown to be weakly collectively optimal, without assuming that voters *infallibly* judge which option is in the common interest. Rather, the weaker assumption that they are just equally better than chance at 'guessing' the common-interest option suffices — if only there are sufficiently large (odd) numbers of voters.

The argument works for any threshold of 'near certainty' that may be proposed and any given (minimal) competence level $c > ½$. For any such threshold and competence level c, one can compute the minimum number n of voters required to achieve the threshold. The lower the competence level, the more voters are needed to reach the threshold. Likewise, for any threshold of near certainty and a given number n of voters, one can compute a minimum level of competence c required to achieve the threshold. The fewer voters there are, the more competent they have to be to reach the threshold.

This latter point emphasises that not only brute numbers but also individual voter competence matters to the quality of democratic outcomes. Thus, the call for raising individual voter competence and thereby improving the democratic input — e.g. through education and deliberation — can be given an important place even in preference-aggregative accounts of democratic decision-making.

	c = 0.51	0.55	0.6	0.75	0.9	0.95
n = 3	0.5150	0.5748	0.6480	0.8438	0.9720	0.9928
9	0.5246	0.6214	0.7334	0.9510	0.9991	0.9999
25	0.5398	0.6924	0.8462	0.9981	0.9999	0.9999
250	0.6241	0.9440	0.9994	0.9999	0.9999	0.9999
1.000	0.7365	0.9993	0.9999	0.9999	0.9999	0.9999
10.000	0.9772	0.9999	0.9999	0.9999	0.9999	0.9999

Table 1

To precisify premise (4) of the above argument, and to get a grasp of how numbers and individual competence levels are correlated with group competence, consider *Table 1* (the entry in each cell gives the probability of a correct majority vote, P_n, for the stated values of n and c).[9]

2.3 Equal-stakes cases with self-interested voters

Now, what happens if we consider equal-stakes cases with *self-interested* voters? These cases differ in some important respects from the usual Condorcet cases with truth-tracking voters and from the above considered equal-stakes cases with common-interested voters. In the latter two scenarios, all voters are assumed to judge the options in the light of an independent standard (whether their desired end is to find the truth or to promote the common interest). Were they all fully competent (that is, certain) in the light of their desired end, and voted accordingly, all would then vote for the same option: the correct one. In these cases, the voters' competence level equals their probability to correctly judge (and vote for) the common-interest option.

In contrast, self-interested voters judge the option in accordance with their respective individual standard of self-interest. Were they all fully competent in the light of their desired end, and voted according to their judgment, some would vote for one option — and others for the other. In such a scenario, we know that the option that is in the self-interest of the majority stake-holders is in the common interest, while the option that is in the interest of the minority stake-holders is opposed to the common-interest option. Hence, given Voting According to Judgment, fully competent majority stake-holders would in effect vote for the common-interest option, while the fully competent minority stake-holders would vote against it. And this means that the latter voters can be described as

[9] Cf. Miller (1986: 176), who also notes that 'most ["0.9999"] entries actually round off to "1.0000" but this is not done to indicate residual uncertainty'.

maximally *in*competent, according to the independent standard of the common good. And likewise, a minority stake-holder who is slightly better than chance at correctly judging her self-interest option can be described as slightly worse than chance, according to the independent standard.

It is at this stage convenient to distinguish two notions of competence. On the one hand, we deal with the voters' competence in judging which option serves their own desired end — be it the promotion of self-interest or of the common interest. Let us call this a voter *i*'s *end-competence*, e_i. On the other hand, what matters to Classical CJT (and the other Condorcet theorems) is the voters' probability to judge correctly according to the independent standard — in our case: to judge correctly which option is in the common-interest. Let us call this a voter *i*'s *CJT-competence*, c_i. Now, a voter's CJT-competence may come apart from her end-competence, namely, whenever she is a self-interested minority member. Such a voter *j*'s CJT-competence equals her end-*in*competence, that is, $c_j = 1 – e_j$. Thus, if she is maximally end-competent, $e_j = 1$, then she is maximally CJT-incompetent, $c_j = 0$.

Note that for any voter *i* who is not a self-interested minority stake-holder, her CJT-competence equals her end-competence, that is, $c_i = e_i$. If she is common-interested, her desired end is the promotion of the common interest, and hence her end-competence equals her CJT-competence. And if she is a majority stake-holder, her self-interest coincides with the common interest, and hence, whatever her desired end among these two, her end-competence equals her CJT-competence.

The connection between the two notions of competence can be illustrated with this simple example. Let us say that all voters in a group are self-interested and fairly competent when it comes to correctly judging which option is in their self-interest, such that for each voter *i*, her end-competence $e_i = 0.75$. Then, each majority member *i* has a CJT-competence of $c_i = 0.75$, as her self-interest coincides with the common interest. Yet each minority member *j* only has a CJT-competence of $c_j = 0.25$, since the option that is in her self-interest is opposed to the common-interest option. We could thus describe the minority stake-holders as worse than chance at judging what is in the common interest — although, and because, they are better than chance at judging what is in their self-interest.[10]

All this means that, if we, as before, assume equal minimal competence and spell this out as equal minimal *end*-competence, the voters' CJT-competence may vary and may be way worse than chance, as in the just considered illustration.[11] That is, for self-interested voters, assuming equal minimal end-competence implies that voters may be heterogeneously and less than minimally (CJT-)competent. This, however, contradicts Classical CJT's assumption of Equal Minimal Competence, which means that the theorem cannot be applied in the present cases.

However, the Condorcet jury theorem has been generalised to cover cases with heterogeneous and less than minimal voter competence. My following argument establishes that such a generalisation does apply to the present cases.

[10] Note that in the following, whenever necessary, I make explicit whether CJT-competence or end-competence are at play. Whenever no such specification is made, e.g. when referring to the Condorcet literature, 'competence' refers to 'CJT-competence'.

[11] I say that CJT-competence *may* vary because there are cases where it does not: namely, when there are no minority members, that is, when one and the same option is in every voter's self-interest. Note that we could simply assume all voters to be equally minimally *CJT*-competent, and then proceed to apply Classical CJT and make the argument from weak collective optimality. This, however, would be an overly artificial assumption since it would imply that all majority stake-holders are equally better than chance and all minority stake-holders equally worse than chance (by the same margin) at judging their self-interest option. It is also an unnecessary assumption, given that we can solve the problem in another way, as shown in the remainder of this section.

Owen, Grofman and Feld show that the asymptotic conclusion holds for heterogeneously (CJT-) competent voters whenever their *average* (CJT-)competence is greater than chance.[12] We can define the following.

> ***Minimal Average CJT-Competence:*** The voters are minimally (CJT-)competent on average, that is, the average of their individual probabilities to judge the correct option (according to the independent standard) as promoting their given end, is above chance, $c^* > ½$.

Then, we can state the theorem as follows.

> ***Heterogeneous Competence CJT:*** For binary decisions with exactly one correct option (according to an independent standard) and a group of $n = 2m+1$ voters, given Minimal Average CJT-Competence, Voting According to Judgment, and Voter independence, the probability of a correct majority vote P_n strictly increases with the number of voters n and approaches certainty as the number of voters increases to infinity.

The following argument shows that this theorem can be applied to the present case of self-interested voters (with equal stakes), if we assume that they are equally minimally end-competent. First, I show that Minimal Average CJT-Competence is implied by the following assumption.

> ***Equal Minimal End-Competence:*** Every voter is equally better than chance at judging the options correctly in the light of her desired end, such that every voter's individual end-competence $e > ½$.

Note that *average CJT-competence*, c^*, is simply the sum of all voters' CJT-competences divided by the number of voters. Let us look at a group of $n>0$ voters who satisfy Equal Minimal End-Competence. Their end-competence is better than chance, which can be expressed as $e = ½+b$ (with $0 < b \leq ½$). Within the group, there is a majority of $m = n/2+a$ voters i (with $0 < a \leq n/2$) whose CJT-competence equals their end-competence (thus, $c_i = ½+b$). Moreover, there is a minority of $n–m = n/2–a$ voters j whose CJT-competence equals their end-*in*competence (this means that $c_j = ½–b$). Hence, the average CJT-competence c^* is:

$$
\begin{aligned}
c^* &= (mc_i+(n–m)c_j)/n \\
&= [(n/2+a)(½+b)+(n/2–a)(½–b)]/n \\
&= (n/4+a/2+nb/2+ab+n/4–a/2–nb/2+ab)/n \\
&= (n/2+2ab)/n \\
&= ½+2ab/n\ .
\end{aligned}
$$

Since a, b, and n all are greater than 0, average CJT-competence c^* must be strictly greater than ½, that is, above chance. This proves that for all here considered equal-stakes cases with self-interested voters, Equal Minimal End-Competence implies Minimal Average CJT-Competence. Hence, given Voting According to Judgment and Voter Independence, Heterogeneous Competence CJT can be applied in the here considered equal-stakes cases with self-interested, equally minimally competent voters.[13]

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[12] Owen, Grofman and Feld (1989: 3f.).

[13] Note that Voting According to Judgment implies, for a self-interested minority stake-holder j with end-competence $e_j = 0.75$, and hence CJT-competence $c_j = 0.25$, that this voter has a 0.75 chance to vote for her self-interest option (as she intended) and a 0.25 risk of voting for the common-interest option instead. And likewise, for a self-interested majority stake-holder i with end-competence $e_i = 0.75$, and hence CJT-competence $c_i = 0.75$, this assumption means that she has a 0.75 chance to vote for her self-interest option (which coincides with the common-interest option), and a 0.25 risk of voting against it by mistake. Thus this assumption does not spell trouble for cases with self-interested voters who vote according to different ends.

This theorem can now be employed as a first premise in the *second argument from weak collective optimality*. The second premise is simply the conclusion of my just stated arguments.

(1) Heterogeneous Competence CJT.

(2) For equal-stakes cases with self-interested voters, given Equal Minimal End-Competence, then Minimal Average CJT-Competence holds.

(3) The correct option is the common-interest option, according to the independent standard of the given criterion of the common good.

(4) For equal-stakes binary decisions, the weighted majority rule selects the option for which a majority votes (since it assigns equal votes to all voters and selects the option which receives a majority of votes).

(5) For any given level of average (CJT-)competence (derived from a set of individual end-competences), there is a range of probability levels of a correct majority vote that we call 'near certainty' and a corresponding range of numbers of voters that we call 'sufficiently large' numbers.

(6) A rule that with near certainty selects the common-interest option, as defined by the given criterion of the common good, is weakly collectively optimal, according to this criterion.

Hence:

(7) For equal-stakes binary decisions with exactly one common-interest option (according to the given criterion of the common good) and a group of a sufficiently large number n = 2m+1 of self-interested voters, given Equal Minimal End-Competence, Voting According to Judgment, and Voter Independence, the weighted majority rule is weakly collectively optimal (according to this criterion).

Thus, for equal-stakes cases with self-interested voters, the weighted majority rule can be shown to be weakly collectively optimal, without assuming that voters *correctly* judge what is in their self-interest. Rather, the weaker assumption that they are just equally better than chance at 'guessing' their self-interest option suffices — if there is a sufficient (odd) number of voters.[14]

[14] The argument in this section has been devised independently of a similar argument by Miller (1986). Miller considers binary decisions for voters with 'conflicting interests' and defines as an independent standard of success for the collective decision that 'the interests of the majority prevail' (1986: 178). He then shows that, for an odd number of $n \geq 3$ voters, if all voters perceive their individual interest with probability $0.5 < p < 1$, then '(i) P'_n [the probability that majority interests prevail] > p* [the expected proportion of the vote in favor of the majority position]; (ii) P'_n increases as n increases; and (iii) $P'_n \rightarrow 1$ as $n \rightarrow \infty$'. (1986: 180). Since Miller's expression '$0.5 < p < 1$' is equivalent to my 'equal minimal end-competence', his 'P'_n' is equivalent to my 'probability of a correct majority vote', and his 'p*' is equivalent to my 'voters' CJT-competence on average', his extension of the Condorcet jury theorem is in fact equivalent to the conjunction of the above Heterogeneous competence CJT and Equal Minimal End-Competence.
I have refrained from replacing my, involving Heterogeneous competence CJT, with Miller's more straightforward theorem. The reason is that I hope that my elaborations around the Equal Minimal End-Competence assumption, in terms of its implications on the group average of CJT-competence (Minimal Average CJT-Competence), which result in the argument's second premise, can make the results more intuitively intelligible.

2.4 Equal-stakes mixed-motivation cases

Thus far, we have considered equal-stakes cases with, we might say, homogeneously motivated voters: all are either assumed to be exclusively common-interested or exclusively self-interested. What happens if there is a mix of self- and common-interested voters in the group? Consider the following series of scenarios with heterogeneously motivated voters.

> ***Only self-interested voters.*** In this equal-stakes binary decision, each voter *i* is self-interested and equally minimally end-competent, with $e_i > ½$. From this latter assumption, we can infer Minimal Average CJT-Competence (as shown in the previous section). Then, as shown by the *second argument from weak collective optimality* above, given Voting According to Judgment and Independence, the weighted majority rule is weakly collectively optimal for sufficiently large (odd) numbers of voters.
>
> ***One common-interested voter.*** Now, we change the above scenario just a bit, such that all voters — except one — are self-interested. This voter, *j*, is instead common-interested and (as before) minimally end-competent with $e_j > ½$. That is, her desired end has changed from self-interest to the common interest, while her end-competence is the same as before. Then, there are two possibilities.
>
> Either (1) i_1 is a majority stake-holder. This means that the option that is in her self-interest is also in the common interest. Then, the present scenario is extensionally equivalent to the previous one, such that we can infer the weighted majority rule's weak collective optimality, given Voting According to Judgment, Independence, and sufficiently large (odd) numbers of voters.
>
> Or (2) i_1 is a minority stake-holder. This means that the option that is in her self-interest is not in the common interest. Then, the present scenario differs from the one above in the following respect. In the *Only self-interested voters* case, since minority stake-holder *j* is self-interested, her CJT-competence equals her end-*in*competence. That is, $c_j = 1–e_j < ½$. Yet in the present *One common-interested voter* case, since she now is common-interested, *j*'s CJT-competence equals her end-competence. That is, $c_j = e_j > ½$, which means that, compared to the previous scenario, *j*'s CJT-competence is now higher. Thus, the group's average CJT-competence must be higher. Thus, in this case, Minimal Average CJT-Competence must still hold.

So even in this case, given Voting According to Judgment and Independence, Heterogeneous competence CJT can be applied. Adding the other premises (3) to (6) from the above *second argument from weak collective optimality*, we can then again infer that the weighted majority rule is weakly collectively optimal, given sufficiently large (odd) numbers of voters.

We can now proceed to *Two common-interested voters*, *Three...*, *Four...* and so on, all the way to *Only common-interested voters*, where all voters are common-interested. Every time, the same kind of reasoning applies. For each case, given Voting According to Judgment and Independence, Heterogeneous Competence CJT can be applied and, adding premises (3) to (6) from the above *second argument from weak collective optimality*, the weighted majority rule can be shown to be weakly collectively optimal for sufficiently large (odd) numbers of voters. This means that the above *second argument* can be adapted to a *third argument from weak collective optimality*, for 'mixed' groups of self- and common-interested voters, such that ultimately the following conclusion is established.

> ***Conclusion:*** For equal-stakes binary decisions with exactly one common-interest option (according to the given criterion of the common good) and a group of a sufficiently large number $n = 2m+1$ of common- or self-interested voters, given Equal Minimal End-Competence, Voting According to

> Judgment, and Voter Independence, the weighted majority rule is weakly collectively optimal (according to this criterion).

Thus, for equal-stakes 'mixed' cases with self-interested or common-interested voters, the weak collective optimality of the weighted majority rule can be established without assuming that voters *correctly* judge what is in their self-interest or in the common interest. Rather, the weaker assumption that they are just equally better than chance at 'guessing' their respective self- or common-interest option suffices — if there is a sufficient (odd) number of voters.

2.5 Mixed cases with heterogeneously end-competent voters

It may now be objected that Equal Minimal End-Competence is still a pretty strong assumption. Could we rebuild the argument from weak collective optimality on the much weaker assumption of *heterogeneous* (and possibly even *worse-than-chance*) end-competence?

Relaxing this assumption comes at the price that we no longer can infer Minimal Average CJT-Competence. Hence, we can no longer apply Heterogeneous Competence CJT, and thus no longer make the *third argument from weak collective optimality*.

However, we can reverse the objection and simply state that *if* Minimal Average CJT-Competence holds, then the above *third argument from weak collective optimality* still shows the weighted majority rule to be weakly collectively optimal, for all equal-stakes cases with self- *or* common-interested voters, with different (and even less-than-minimal) end-competences, given a sufficient (odd) number of voters, and given the additional assumptions of Voting According to Judgment and Voter Independence.

This argumentative move may seem rather uninformative, at first glance. Surely we would want to know whether there are any further reasons to believe that Minimal Average CJT-Competence holds. But as it stands, this would have to be determined independently for any given group of voters. Still, this line of thinking has at least two advantages.

First, it effectively removes the need for any specific assumption concerning voter motivation: voters may be motivated to vote according to any common-, self-, or partial-interested (see section 1 above) desired end. As long as their individual CJT-competence — as calculated from their individual end-competence in the light of their specific end — does not bring down average CJT-competence to ½ or below, the *third argument from weak collective optimality* applies.

Second, our current analysis may e.g. help us better understand certain scenarios, such as the following Mixed motivation case. Jonathan Wolff (1994) uses this case to contest the collective optimality of majority rule. Although Wolff's criticism targets the *simple* majority rule, recall that the weighted majority rule is extensionally equivalent to the simple majority rule in equal stake cases (as currently considered). Thus, Wolff's objection, if sound, should challenge my present arguments as well. Wolff states that 'it can easily be demonstrated that, if part of the electorate vote in pursuit of their own interests, and part for common good [defined as majority interest], then it is possible to arrive at a majority decision which is neither in the majority interest, nor believed by the majority to be for the common good'.[15] Wolff establishes this conclusion with an example, which I restate as follows.

[15] Wolff (1994: 194).

Mixed motivation. There is a group facing two options, *x* and *y*. *x* is in the self-interest of 40% of the group (I call this the first subgroup), while *y* is in the self-interest of 60% (the second subgroup). Moreover, 80% of the entire group (across the first and second subgroup) believe *y* to be the common good, defined as majority interest, while 20% believe this of *x*. Now, suppose that all voters within the first subgroup vote according to their self-interest, while all the voters within the second vote according to their perception of the common interest. Then, 52% will vote for *x* — the option that is *not* in the majority interest and *not* believed to be so by a majority of the group. (This can be easily seen, as all in the first subgroup — 40% of the group — will vote for *x*, as well as the 20% of those within the second subgroup — constituting 60% of the group — who take *x* to be the common good, which amounts to an additional 12%.)

Wolff takes this to show that a mixed motivation assumption undermines the optimality of majority rule. However, according to my present analysis, we should rather focus on the competence assumptions at play. There are 40% minority stake-holders *i* who are self-interested and maximally end-competent, with $e_i = 1$. This gives them a CJT-competence of $c_i = 0$. Moreover, there are 60% majority stake-holders who are common-interested. Among these, an 80% subset of voters *j* are maximally end-competent, with $e_j = 1$, such that their CJT-competence is $c_j = 1$, and a 20% subset of voters *k* are maximally end-incompetent, with $e_k = 0$, such that their CJT-competence is $c_k = 0$. This means that the group's average CJT-competence $c^* = 40\%\cdot0+60\%\cdot80\%\cdot1+60\%\cdot20\%\cdot0 = 0.48$. As this is well below chance, the poor result should not come as a big surprise.

The analysis suggests that one driver of Wolff's objection is voter incompetence. In fact, it can be shown that this is *all* there is to the problem, i.e. that the mixed motivation assumption is not relevant at all for Wolff's conclusion. To see this, consider the following case.

Homogeneous (self-interested) motivation. There is a group facing two options, *x* and *y*. *x* is in the self-interest of 40% of the group (the first subgroup), while *y* is in the self-interest of 60% (the second subgroup). Now suppose that all are motivated by their self-interest. Within the first subgroup, all correctly perceive *x* to be in their self-interest. Yet within the second subgroup, 20% mistakenly judge *x* to be in their self-interest. All vote accordingly. Then, 52% will vote for *x* — the option that is *not* in the majority interest. (This can be easily seen, as all within the first subgroup — 40% of the group — will vote for *x*, as well as the 20% of those within the second subgroup — constituting 60% of the group — who mistakenly judge *x* to be in their self-interest, which amounts to an additional 12%.)

This scenario brings out that it is solely the assumption of *voter incompetence* that drives Wolff's conclusion that 'it is possible to arrive at a majority decision which is [not] in the majority interest'.[16] My above analysis brings out in further detail how voter incompetence works, in the original Mixed motivation case.

Let us return to our present case for the weighted majority rule under less than full voter competence. Until now we have only considered equal-stakes cases, in order to simplify the investigation. The next question is whether we can broaden the attained results to cover unequal-stakes cases as well.

[16] Wolff (1994: 194). (Note that we could easily add the assumption that 80% of the entire group (across minority and majority stake-holders) believe *y* to be the common good to also derive the second part of Wolff's conclusion that the simple majority rule selects an option *not* 'believed by the majority to be for the common good.') For a similar analysis of Wolff's problem, see Graham (1996).

2.6 Unequal-stakes cases and the independence assumption

There is one seemingly promising strategy to straightforwardly apply all of the above results to unequal-stakes cases, in which voters may have different numbers of votes. The suggestion is to simply replace the term 'voter' with the term 'vote' in the above arguments, or, to put it differently, to treat every vote the weighted majority rule assigns in unequal-stakes cases as if it were a separate voter.

To illustrate this suggestion: assume that there are three voters, i_1, i_2 and i_3, and that i_1's stakes are three times as high as i_2's and i_3's, respectively. The weighted majority rule then assigns, say, three votes to i_1 and one vote each to i_2 and i_3. Thus, there are five votes distributed among the three voters. We could now treat this case just as a case with five voters, where each voter has one vote, and proceed with the argument as before.

In other words, the suggested strategy is that we restate the above conclusion of the *third argument from weak collective optimality* in terms of *votes* rather than *voters*, since what matters for the weighted majority rule in unequal-stakes cases is the number (majority) of votes, rather than of voters. That is, we do not focus on common- or self-interested voters *i*, who cast their votes for the intended option with any end-competences e_i, such that their average CJT-competence $c^* > ½$. Instead we focus on votes *v*, that are cast according to the voters' judgment with any end-competences e_i, such that the votes have an *average probability* $p^* > ½$ of being cast for the common-interest option. We can thus define:

> ***Minimal Average CJT-Probability:*** All votes are cast according to the voters' judgments, with any end-competences, such that the votes' average probability of being cast for the common-interest option, p^*, is above chance, $p^* > ½$.

We then also have to redefine Voter Independence. In analogy to my characterisation of *voter* independence (see 2.1 above), independence between votes can be spelled out in terms of that *vote* v_1's probability of being cast for the correct option equals its conditional probability of being so cast, given vote v_2's probability of being cast for the correct option. Alternatively, the probability that both v_1 and v_2 are simultaneously cast for the correct option equals the product of their individual probabilities. Let us then define:

> ***Vote Independence:*** Every vote is cast probabilistically independently, that is, how each vote is cast does not depend on how others are cast.

Then we could suggest the following.

> ***Suggested Conclusion:*** For unequal-stakes binary decisions with exactly one common-interest option (according to the given criterion of the common good) and a group of a sufficiently large number $n = 2m+1$ of votes that are cast by any number $l \leq n$ of voters, given Minimal Average CJT-Probability and Vote Independence, the weighted majority rule is weakly collectively optimal (according to this criterion).[17]

However, we can now see that the Suggested Conclusion runs into a big problem. Recall that the weighted majority rule assigns to each voter a number of votes, as an *indivisible* vote bundle. But then,

[17] Note that Voting According to Judgment is implied by Raised Average CJT-Probability, as the latter states that votes are cast according to the voters' judgment.

if some voters have indivisible vote bundles with more than one vote — as would be the case with unequal stakes — Vote Independence does not hold.

To illustrate the problem: say that we know that common-interested voter i_1 is given an indivisible bundle of two votes, and that she is minimally end-competent. We can infer that each of her two votes is slightly more likely than not to be cast in the common interest. But if we learn that one of i_1's votes has been in fact cast in the common interest, then we can infer a change in probability, namely that i_1's other vote has been cast in the common interest *with certainty*. In other words, the probability for one of i_1's votes v_1 to be cast in the common interest does *not* equal v_1's conditional probability of being cast in the common interest, given that v_2 is cast in the common interest. Thus, i_1's votes v_1 and v_2 are not independent.

How damaging is such a violation of the independence condition? Before we can answer this question, we need to be clear about which kind of independence is violated in unequal-stakes cases, and which kind is assumed in the stated Condorcet theorems.

In the literature on the Condorcet jury theorem, violations of the independence assumption are usually assessed in terms of *positive* probabilistic correlation of judgments or votes: that is, that a pair of judgments or votes is *more* likely to concur than probabilistically expected.[18] Such correlation may be due to causal connections between the votes themselves — let us call this *direct causal dependence* — or between the votes and other factors that function as common causes — let us call this *common cause dependence*.[19] Direct causal dependence between votes may, e.g., result when the vote of a ‘voting leader’ influences the votes of other voters.[20] Common cause dependence may, e.g., result when voters take part in mutual deliberation, or belong to the same ‘schools of thought’, when such partaking or affiliation brings them to perceive things in similar ways.[21] Another common cause of probabilistic correlations between votes is commonly shared evidence.[22] Even seemingly irrelevant factors, such as the weather, might work as a common cause. Sunny skies, for instance, might bring voters to look more optimistically at a pair of options and to underestimate their harmful effects.[23]

The independence assumption of all the above stated Condorcet theorems serves to rule out any positive correlation between judgments or votes — whether due to direct causal dependence or to common cause dependence. Why is it important to rule this out? The following extreme example gives a simple illustration of what happens when the assumption is violated.

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[18] Correlation between judgments or votes may also be *negative*, namely, when a pair of judgments or votes is *less* likely to concur than probabilistically expected. I start out by focusing on positive correlation, as arising from indivisible vote bundles. I do, however, return to negative correlation in the next section.

[19] There is no need to presuppose a specific theory of causation here. My arguments for the weighted majority rule presuppose only the presence (or absence) of probabilistic correlation between voters or votes. We can then, as here suggested, make sense of such correlation by referring to the causal relations between voters and votes. Yet the question of how to understand the nature of these causal relations is of no direct significance for my arguments.

[20] Boland (1989: 185). Cf. even Grofman, Owen and Feld (1983) and Hawthorne (*mimeo*).

[21] Ladha (1992: 618).

[22] Dietrich and List (2004). Note that the various common causes I list may differ in their effects. E.g. belonging to different ‘schools of thought’ might bring people to judge an issue differently, even though they share the same evidence. Or taking part in mutual deliberation might bring two people to judge similarly, even though each has separate evidence that contradicts the other's.

[23] Cf. Dietrich and Spiekermann (2010: 5). For a good discussion of these sources of correlation and of different independence conditions, see Dietrich and Spiekermann (2010).

Renowned authority. A group of three voters is on average moderately competent, with $c^* = 0.7$. Were all to vote according to their independent judgments, the probability of a correct majority vote would exceed 0.7. However, one of the voters, i_1, is a renowned authority within the group, yet she is in fact only minimally competent with $c_1 = 0.6$. Being renowned as she is, her judgment determines all the other voters' judgments. Since they vote according to their authority-determined judgments, the majority of the voters (in fact, all of them) will be no more likely to vote for the correct option than the renowned authority. Even if more voters were added to the group, as long as they voted according to their authority-determined judgments, the probability of a correct majority vote would not increase. The asymptotic conclusion does not hold.

The Renowned authority case is an extreme illustration of the damaging effect of violating the independence assumption through positive correlation (due to direct causal dependence between voter judgments). Without Independence, Heterogeneous Competence CJT does not hold.

It needs to be pointed out that the independence assumption is not designed to rule out *any* positive correlation between votes (or judgments; I henceforward focus on votes). It does not require *unconditional* independence (unconditionally uncorrelated judgments or votes). In fact, an assumption of unconditional independence would conflict with the relevant competence assumption. Consider a typical Condorcet context, with correctness-tracking voters who vote according to their judgments, and a correct option (according to some independent standard). Suppose that we know that all voters have a competence (probability to judge correctly) of $c = 0.75$. Unbeknownst to us, the correct option is *x*. Then, when 999 voters have cast their votes, and we have observed a large majority of them to be cast for *x*, we can infer that, quite likely, *x* is the correct option. Hence, we can infer that, quite likely, the 1.000th voter will cast her vote for *x* with a 0.75 probability. This means that the votes are probabilistically correlated, in virtue of the assigned competence (correctness-tracking probability). They are not unconditionally independent. Yet they can still be independent in another sense, namely, *state-conditionally* independent. This can be spelled out as follows. Say that we know that *x* is the correct option — let us call this the state of the world. Knowing that a large proportion of the 999 votes is cast for *x* will tell us nothing new about the probability that the 1.000th voter will vote for *x*. This is so since, knowing the state — that *x* is the correct option — and knowing that everyone has a competence of $c = 0.75$, we can infer independently that the 1.000th voter will vote for *x* with a 0.75 probability. In other words, the state-conditional probability of any voter *i*'s vote to be cast for *x*, equals her state-conditional probability, given that some other voter *j* voted for *x*. *i*'s and *j*'s votes are then probabilistically uncorrelated, conditional on the state.[24]

The independence assumption of *Classical CJT*, as well as the other previously considered Condorcet theorems, presupposes state-conditional independence. This assumption is violated when the votes are probabilistically correlated *given the state*, e.g., when 999 votes still tell us something new about the 1.000th voter's probability to vote for *x*, even though we know the state of the world and voter competence. As stated, such correlations between votes can be due to, e.g., commonly shared distorting evidence, or the presence of a renowned authority whose vote determines how others vote.

Now, let us return to our here considered context with voters who have unequal stakes in a decision. Consider a case where one voter, *i*, has an indivisible bundle of several votes. Then, even if we conditionalise on the state of the world — if we learn that *i* casts one of her votes, v_1, for *x*, we will

[24] In a way we might say that the state of the world is a common cause, since it effects correlation between votes (cf. 5.2.8 below). Still, in order to avoid confusion, in the following text, I do not include the state within the class of common causes.

change the probability we assign to another one of her votes, v_2, of being cast for x, to certainty.[25] This means that state-conditional independence is violated when we introduce indivisible vote bundles. Our present case is thus in a relevant way analogous to the considered *Renowned authority* case (it would be entirely analogous were i the only voter), in which the asymptotic conclusion no longer holds. All the above arguments for collective optimality rely on this conclusion. Without it, the above suggested strategy — to simply and straightforwardly expand this argument, from equal-stakes cases to the domain of unequal-stakes cases — fails.

In the following section, I explore how damaging this violation of the independence assumption is for the case for the weighted majority rule. Note that in this paper, I assume common-cause independence. That is, I assume that votes are not probabilistically correlated due to any common causes (besides the state of the world, that x (or y) is the common-interest option). However, since common-cause independence — due to, e.g., shared evidence — is still a pretty strong assumption in its own right I return to it in 2.8 and see whether it can be relaxed as well.

2.7 Unequal-stakes cases and direct causal dependence

We have just noted that direct causal dependence of votes in unequal-stakes cases is relevantly analogous with that in the rather extreme *Renowned authority* case, in which one voter's judgment determines the way all others judge and vote. Let us now consider a small series of examples with dependent voters that are a little less extreme, to get a better understanding of the effects of direct causal dependence between *voters.* (I get back to dependence between *votes* at the end of this section.)

I first describe an (equal-stakes) case without direct causal dependence and then transform it into a case where there is such dependence due to voting leaders.

> ***Uncorrelated voters*.** There are three voters in total, with one vote each. We assume their votes to be (state-conditionally) uncorrelated. We can interpret this as there being three voters who vote independently: they have separate evidence, no communication, no 'schools of thought', voting leaders, or the like. While voter i_1 is highly competent, with p_1 = 0.9, i_2 and i_3 are less competent, with p_2 = 0.41 and p_3 = 0.4 (see *Table 2*).

	i_1	i_2	i_3
c_i	0.9	0.41	0.4

Table 2

From these figures we can calculate the voters' average competence as c^* = 0.57, better than chance. Now, we can see that the correct option will be selected by weighted majority rule under only two specific sets of circumstances: (1) either all three voters vote correctly, or (2) exactly two of them do. (1) occurs if and only if i_1, i_2, and i_3 vote correctly. The probability for this is 0.9·0.41·0.4 = 0.1476. (2) occurs if and only if (a) i_1 and i_2 vote correctly and i_3 votes incorrectly, the probability for this is 0.9·0.41·0.6 = 0.2214; (b) i_1 and i_3 vote correctly and i_2 votes incorrectly, the probability for this is 0.9·0.59·0.4 = 0.2124; or (c) i_1 votes incorrectly, while i_2 and i_3 vote correctly, the probability for this is

[25] To be precise, there will be a change in probability only if i_1 is assumed to be less than fully competent.

0.1·0.41·0.4 = 0.0164. In sum then, the probability that the majority votes correctly equals P_n = 0.5978 (a figure that is higher than the voters' average competence).

Now, let us see what happens if we introduce positively correlated voters.

> ***Positively correlated voters***. We introduce two additional voters, i_4 and i_5, according to *Table 3*. These two additional voters are 'followers' to a pair of 'voting leaders'. More specifically, i_4 votes with i_1 (with certainty), while i_5 votes with i_3 (with certainty); they thus mirror their respective voting leader's probability to vote for the correct option. Thus, these two pairs are maximally positively correlated. The other eight pairs of voters, we assume, are (state-conditionally) uncorrelated.

	i_1	i_2	i_3	i_4	i_5
c_i	0.9	0.41	0.4	0.9	0.4

Table 3

What are the effects of introducing such positive correlation on the probability that a majority votes for the common-interest option? Introducing these new voters raises the voters' average competence, c^* = 0.602. However, as can be seen from the following short argument, it does *not* raise the probability that the majority votes correctly.

In the present case, the majority rule will select the correct option if either (1) all five voters vote correctly, or (2) exactly four or (3) exactly three vote correctly. (1) occurs if and only if i_1 (and hence i_4), i_2, and i_3 (and hence i_5) vote correctly. The probability for this is 0.9·0.41·0.4 = 0.1476. (2) occurs if and only if i_1 (and hence i_4) and i_3 (and hence i_5) vote correctly and i_2 votes incorrectly. The probability for this is 0.9·0.59·0.4 = 0.2124. Finally, (3) occurs if and only if (a) i_1 (and hence i_4), and i_2 vote correctly and i_3 (and hence i_5) vote incorrectly, the probability for this is 0.9·0.41·0.6 = 0.2214; or (b) i_1 (and hence i_4) vote incorrectly, while i_2 and i_3 (and hence i_5) vote correctly, the probability for this is 0.1·0.41·0.4 = 0.0164. Summing up these probabilities, the probability that the majority votes correctly again equals P_n = 0.5978 – just as in the three voter case. This illustrates that the asymptotic conclusion is in trouble, when there is direct causal dependence between voters.[26] Increasing the number of voters, n, does not strictly increase the probability of a correct majority vote, P_n.

It might now be objected that the calculations for the *Positively correlated voters* case are inconsistent. For calculating average competence, c^*, we treated it as a case with five voters, yet for calculating P_n we treated it as a three-voter case. However, this objection is off the mark. We are, of course, dealing with a five-voter case, so both c^* and P_n should be calculated for the group of five. Yet, when it comes to calculating P_n, our information about direct causal dependence between voters tells us that certain distributions of votes are rendered impossible — namely, the ones where the respective voting leader and follower vote against each other. This changes the probability *that a majority votes correctly* throughout, such that it resembles the three-voter case (as specified in *Table 1*). Yet this does not mean that we are in fact dealing with a three-voter case. There are five voters, and each voter's

[26] I formulate this a bit cautiously as a suggestion, since by adding the two "followers" i_2 and i_5, the group's average competence rises from c^* = 0.57 to c^* = 0.602. This violates a potential condition of "fixed" average competence. It is remarkable, however, that even when average competence is *raised* by adding more voters, the probability of a correct majority vote may *not* increase, when there is direct causal dependence between voters.

competence, as the probability *that this individual voter votes correctly*, must be taken into account when calculating the group's average competence c^*.

The just considered two cases illustrate that positive correlation between voters may reduce the probability of a correct majority vote. This challenge is dealt with, on a more general level, by another Condorcet theorem, which holds for dependent voters. Ladha shows that the asymptotic (and non-asymptotic) conclusions hold even when votes are correlated — e.g. due to a voting leader — if certain other conditions are satisfied.[27]

Let us call the probability that two different voters *i* and *j* both vote simultaneously for the correct option *simultaneity-probability*, s_{ij}. s_{ij} is defined as a function of *i*'s and *j*'s competences (their individual probabilities to vote for the correct option) and the coefficient of correlation between their votes. When *i*'s and *j*'s votes are *uncorrelated*, their simultaneity-probability s_{ij} equals the product of their competences, that is $s_{ij} = c_ic_j$. When *i*'s and *j*'s votes are *positively correlated*, their probability to simultaneously vote for the correct option is higher than probabilistically expected from their individual competences, that is, $s_{ij} > c_ic_j$. When they are maximally positively correlated, such that, e.g., *j* is certain to vote with *i*, s_{ij} equals *i*'s competence, that is $s_{ij} = c_i$. This is the maximum simultaneity-probability level. Finally, when *i* and *j* are *negatively correlated*, their probability to simultaneously vote for the correct option is lower than probabilistically expected from their individual competences, that is, $s_{ij} < c_ic_j$. And when they are maximally negatively correlated, such that, say, *j* is certain to vote against *i*, their simultaneity-probability level is at its minimum $s_{ij} = 0$.[28]

From the individual simultaneity-probabilities s_{ij} we can calculate an average s^* for all pairs of *n* voters to vote simultaneously for the correct option. To do that, we sum the s_{ij} for every pair of voters *i* and *j* and divide it by the total number of pairs in the group ($\frac{1}{2}n(n-1)$). E.g., for three voters i_1, i_2, and i_3, the average simultaneity-probability $s^* = (s_{12}+s_{13}+s_{23}) / \frac{1}{2}n(n-1) = (s_{12}+s_{13}+s_{23}) / 3$. We can now define the following.

> ***Simultaneity-Probability Condition:*** The voters' average simultaneity-probability is smaller than a specific function *f* of the numbers of voters *n* and their average competence c^*, such that $s^* < f(n, c^*)$, with $f(n, c^*) = c^*-n/(n-1)\cdot[(c^*-1/4)(1-c^*)]/c^*$.

With this terminology in place, we can state Ladha's theorem as follows.[29]

> ***Voter dependence CJT:*** For binary decisions with exactly one correct option (according to some independent standard) and a group of $n = 2m+1$ voters, given Minimal Average CJT-Competence and Voting According to Judgment, the probability of a correct majority vote P_n strictly increases

[27] Ladha (1992).

[28] To be precise, $s_{ij} = \rho_{ij}\sigma_i\sigma_j+c_ic_j$, where ρ_{ij} is the coefficient of correlation between *i*'s and *j*'s votes, $\sigma_i^2 = c_i(1-c_i)$ is the variance of voter *i*'s votes, and c_i is, as before, voter *i*'s competence level (such that $1-c_i$ is her incompetence level) (cf. Ladha 1992: 625).
This equation can be made intelligible as follows: when *i*'s and *j*'s votes are *uncorrelated*, the coefficient of their correlation ρ_{ij} is zero, and so s_{ij} equals the product of their competences, that is $s_{ij} = c_ic_j$. When *i*'s and *j*'s votes are *positively correlated*, ρ_{ij} is greater than zero, and so $s_{ij} > c_ic_j$. When they are maximally positively correlated, such that, say, *j* is certain to vote with *i*, $\rho_{ij} = 1$, and s_{ij} equals *i*'s competence, that is $s_{ij} = c_i$. This is the maximum simultaneity-probability level. Finally, when *i* and *j* are *negatively correlated*, ρ_{ij} is smaller than zero, and $s_{ij} < c_ic_j$. When they are maximally negatively correlated, such that, say, *j* is certain to vote against *i*, $\rho_{ij} = -1$, and $s_{ij} = 0$. This is the minimum simultaneity-probability level.

[29] Ladha (1992: 626, 628).

with the number of voters *n* and approaches certainty as the number of voters increases to infinity and as average simultaneity-probability s^* approaches the squared average competence, c^{*2}.

This theorem differs from all the previous theorems in that it does not rely on an assumption of independence. Yet we should stop to note that its states the asymptotic conclusion conditional on a further assumption. This conclusion states that — given Minimal Average CJT-Competence and Voting According to Judgment — the probability of a correct majority vote approaches certainty as the number of voters increases to infinity *and* as s^* approaches c^{*2}. So now, large enough numbers are not quite sufficient; in addition, s^* must be sufficiently close to the squared average competence c^*.

To get a more intuitive grasp of what this latter clause means, consider the following simplified case (which is a typical case for Classical CJT). There are *n* voters with equal minimal competence *c*, and thus average voter competence $c^* = c$. Moreover, all voters vote independently. That is, all voter pairs are uncorrelated. This means that for each voter pair *i* and *j*, their simultaneity-probability $s_{ij} = c^2$. Thus, the average simultaneity-probability $s^* = c^2$. We can then infer that in such a case with uncorrelated voters, $s^* = c^{*2}$. When positive correlation between voters — e.g. due to voting leaders — is introduced in this case, s^* increases such that $s^* > c^{*2}$. The effect of such an increased s^* on the probability of a correct majority vote, P_n, has been illustrated by my above Positively correlated voters example: P_n in an assumed five-voter case equals P_n in a three-voter case. This suggests that an *increase* in positive correlation has the same effect as a *decrease* in numbers of voters.[30] And thus, an increasing level of s^* can counter the effects of increasing numbers *n*. Thus, when the numbers *n* approach infinity *and* average simultaneity-probability s^* approaches the level it takes without voter correlation (c^{*2}), P_n increases and approaches certainty. In other words, the closer s^* is to c^{*2}, the closer a case with dependent voters resembles a case with independent voters, for which the asymptotic conclusion has been shown to hold.

Now, the discussion in this section has been conducted under the assumption that there is direct causal dependence between *voters* due to there being other voters who function as voting leaders. We may call this form of dependence *interpersonal* direct causal dependence. Yet this study is concerned with dependence between the *votes* held by one and the same voter, whenever she is assigned an indivisible vote bundle. We may thus call this *intrapersonal* direct causal dependence. The previous discussion and the Voter Dependence CJT are of relevance for both forms of dependence. Their effects are the same whether the 'voting leader' is another voter or, metaphorically speaking, just another vote within the same bundle of votes. Thus, all the above results can be straightforwardly applied to unequal-stakes cases with indivisible vote bundles.

We can then restate the above Voter Dependence CJT in terms of votes rather than voters. That is, we state the theorem for a set of $n = 2m+1$ *votes*, which are cast by some number of voters *l* (with $l < n$), who are minimally (CJT-)competent on average. For this set of votes we can calculate an average simultaneity-probability s^*, as the average of the probability that a pair of *votes* is cast simultaneously

[30] This suggestion is supported by Hawthorne's (*mimeo*) results. Hawthorne shows that 'when the average individual competence level is at least slightly better than chance, the more independently people vote, the higher the probability that the majority will select the better policy' (*mimeo*: 23). Thus, although a high level of independence improves the result, complete independence is not a requirement for the theorem to apply. In fact, Hawthorne observes, for lower degrees of independence, 'the effect is merely to lower the group competence level to that enjoyed by a somewhat smaller group of more nearly independent voters' (*mimeo*: 34). For another CJT-extension covering cases with dependent voters, see Boland (1989: 185f.). Boland, however, assumes equal competence and equal voter correlation for all individuals.

for the correct option, for all possible pairs. The thus adapted Vote Dependence CJT can then serve as the first premise in the *fourth argument from collective optimality*.

> **(1)** *Vote Dependence CJT*. For binary decisions with exactly one correct option (according to some independent standard) and set of $n = 2m+1$ votes cast by any number of voters $l < n$, given Minimal Average CJT-Competence and Voting According to Judgment, the probability of a correct majority vote P_n strictly increases with the number of votes n and approaches certainty as the number of votes increases to infinity and as average simultaneity-probability s^* approaches the squared average competence c^{*2}.
>
> **(2)** The correct option is the common-interest option, according to the independent standard of the given criterion of the common good.
>
> **(3)** *The Weighted Majority Rule:* For all individuals and any decision with two options, (a) every individual is assigned a number of votes in proportion to her stakes, and (b) the option that receives a majority of votes is selected as outcome.
>
> **(4)** For any given level of average (CJT-) competence, there is a range of probability levels for a correct majority vote that we call 'near certainty' and a corresponding range of numbers of voters (according to the Condorcet jury theorem) that we call 'sufficiently large numbers' of voters, in combination with a range of average simultaneity-probability s^* sufficiently close to c^{*2} that we call 'on average tolerably correlated votes'.
>
> **(5)** A rule that with near certainty selects the common-interest option, as defined by the given criterion of the common good, is weakly collectively optimal, according to this criterion.
>
> Hence:
>
> **(6)** For binary decisions with exactly one common-interest option (according to the given criterion of the common good) and set of a sufficiently large number $n = 2m+1$ of on average tolerably correlated votes cast by any number $l < n$ voters, given Minimal Average CJT-Competence and Voting According to Judgment, the weighted majority rule is weakly collectively optimal (according to this criterion).

This shows that Condorcet theorems can be applied to the workings of the weighted majority rule, even in cases with indivisible vote bundles due to unequal stakes, where there is direct causal dependence between votes. If the number of votes is sufficiently large, and they are on average tolerably correlated, the weighted majority rule can be shown to be collectively optimal even with less than full voter competence.[31]

Note that, again, employing Minimal Average CJT-Competence in effect removes the need for any specific assumptions about the voters' desired ends. As long as their individual CJT-competence — as calculated from their individual end-competence — does not bring down average CJT-competence to the limit of ½ or below, it does not matter whether voters are motivated to vote according to common-, self-, or partial-interested ends.

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[31] Someone might wish to suggest an easy way of improving group competence: just give everyone more votes — e.g., multiply every voter's stake-proportional number of votes by 100, or why not 1.000.000. However, this suggestion does not work: all these large vote bundles would increase average vote correlation and hence counteract increasing group competence.

In an unequal-stakes case with many votes tied up in indivisible vote bundles, average simultaneity-probability s^* may easily be high, and thus not sufficiently close to the squared average competence c^* to guarantee the weighted majority rule's weak collective optimality. Consider a further twist: a voter's end-competence level and the amount of stakes she holds might be correlated.[32] Plausibly, the more a voter knows to have at stake in a decision, the more motivated she may become to inform herself about the options and their impact on her self-interest and thus the more end-competent she would tend to be. Then, ceteris paribus, if voter *i* had higher stakes in one decision than in another, and thus a larger vote bundle, there would be a higher level of interdependence in the former than in the latter, with lower group competence as a result. However, if *i* in both cases were a majority stake-holder, her higher stakes in the first decision would tend to 'make' her more CJT-competent, such that average group competence might in fact be higher than in the second decision. Thus, the damaging effects of a higher average simultaneity probability s^* for the group might be mitigated by an increase in average competence c^*. Unfortunately, if *i* instead were a minority stake-holder, her higher stakes in the first decision would tend to 'make' her less CJT-competent (because more end-competent), such that average competence might in fact be much lower than in the second decision. Thus, such a correlation between a voter's stakes and her end-competence might be either beneficial or, really, quite harmful, from a common-good perspective.[33]

Of course, there are other factors that may further increase positive correlation between votes. Imagine, for instance, a group of common-interested voters who all share the same misleading evidence as to what option is in the common interest. The misleading evidence works as a common cause, making it more likely for voters to simultaneously vote *against* the common-interest option. Indeed, such *common-cause dependence* is a concern for many theorists who have worked with Condorcet theorems.[34] Addressing these additional difficulties, however, must be postponed to another occasion.

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[32] I owe this suggestion to Gustaf Arrhenius.

[33] One could, of course, question the initial assumption that greater stakes tend to motivate voters to get informed, with the Downsian problem of information costs (Downs 1957).

[34] See e.g. Rawls (1999: 315), stating that "it is [...] clear that the votes of different persons are not independent. Since their views will be influenced by the course of the discussion, the simpler sorts of probabilistic reasoning do not apply". Cf. Estlund (2008: 225f.) Cf. also Dietrich (2008: 10f.) who argues that when the decision problem faced by the voters is 'randomly drawn from a reference class of relevant problems', rather than a specific or 'fixed' problem, the independence assumption is 'usually violated': there are usually common causes, such as shared evidence, which will lead to correlation between votes. Independence could be vindicated by conditionalising on all common causes, but plausibly common causes will vary over the problems within the considered reference class. In order to conditionalise, we need to focus on a 'fixed' decision problem, with its specific common causes. Dietrich also argues, when considering fixed decision problems, we 'can usually not know [...] whether the voters are competent', since this requires knowledge of both the correct option and the nature of the specific common causes (2008: 7f.), and this, Dietrich's contention goes, is something of a tall order. Dietrich's conclusion is thus the following dilemma: in order to vindicate independence, we usually have to sacrifice our faith in the competence assumption, and in order to vindicate the competence assumption, we usually have to sacrifice our faith in the independence assumption. (Note that the title of Dietrich's paper puts the point much more strongly; I think it should be modified to state that 'The premises of Condorcet's Jury Theorem are not [*Usually*] Simultaneously Justified'.) Dietrich and Spiekerman (2011) offer a way out: they restate the independence and competence assumptions (and hence their entire Condorcet theorem) conditional on "the problem": the state *in conjunction with all common causes*. However, their theorem cannot be employed to make a case for the weighted majority rule, since it assumes that voters are independent conditional on the problem. In unequal stakes-cases, with indivisible vote bundles, there is positive correlation between votes even when we have conditionalised on the state and all common causes. As Dietrich and Spiekermann (2011: 21) explicitly point out, direct causal

3 Conclusions

The arguments in this paper employed a number of Condorcet theorems to build a new case for the weighted majority rule, for binary decisions with less than fully competent voters. From the outset, voters were assumed to have their self-interest or the common interest as their desired end, to have correct beliefs regarding which option promotes their respective end, and to vote accordingly. The strong assumption of full voter competence was then relaxed to allow certain better-than-chance competence levels.

Note that the results I derived in this paper apply to odd-numbered groups of voters. As stated in 2.1 above, these results can be extended to even-numbered groups.[35] I state the qualifying term 'odd' in the summary of my results below in parenthesis to account for its being inessential to the argument.

For equal-stakes cases with common-interested voters, the *first argument from weak collective optimality* established that the weighted majority rule selects the common-interest option with near certainty when the strong assumption of full voter competence is replaced by an assumptions of equal minimal (CJT-)competence and, in addition, (state-conditional) independence is assumed between voter judgments, given a sufficiently large (odd) number of voters. For equal-stakes cases with self-interested voters, the *second argument* established the weighted majority rule's weak collective optimality conditional on the assumptions of equal minimal *end*-competence and independence, again given a sufficiently large (odd) number of voters. The *third argument* showed, by a series of steps from *Only self-interested* to *Only common-interested voters* cases that the same conclusion holds for equal-stakes mixed-motivation cases. I moreover addressed and refuted Wolff's alleged Mixed motivation problem within my thus developed framework.

For unequal-stakes cases with self- or common-interested voters, I showed that the assumption of independence is violated, since votes within indivisible vote bundles are positively correlated. However, *the fourth argument* showed that the weighted majority rule is weakly collectively optimal, conditional on minimal average CJT-competence, given a sufficiently large (odd) number of on average tolerably correlated votes. Formulating the condition in terms of average CJT-competence moreover removes the need for any specific motivating assumption: voters may be common-, self- or partial-interested, as long as their individual end-competences result in a sufficiently high average CJT-competence.

Conducting the argument in these steps has had the advantage that we could accurately see which prices we have to pay to adapt the argument from weak collective optimality to different contexts — from equal-stakes common-interested cases to unequal-stakes cases with mixed motivation. For all of these cases, full voter competence, concerning which option is in accordance with the voters' respective desired end, is not required for a compelling case for the weighted majority rule. Overall, the above strong Competence assumption can be relaxed, at the cost of settling for the weak (rather than strong) collective optimality of the weighted majority rule.

All these results depend heavily on the condition that there are sufficiently many voters (or votes). This means that within the traditional political domain of elections and referenda, with large

dependence between voters violates the assumption of problem-conditional independence. This means that their theorem, as it stands, cannot be applied to my cases.

[35] See footnote 6 above for a precisification of this claim.

electorates, the weighted majority rule emerges as a promising candidate from a common-good perspective.

To conclude, my arguments in this paper show that Condorcet theorems are applicable not only within judgment-aggregative accounts of democracy, but can be a useful tool even within other approaches. The classical Condorcet jury theorem, as Ladha puts it, 'assumes that the members of a group who choose between a pair of alternatives (a) share a common goal, (b) vote (statistically) independently, and (c) vote for the better alternative with a probability greater than 0.5'.[36] Recent literature in this field has developed our understanding as to how far assumptions (b) and (c) can be relaxed in order for some version of the Condorcet-results to hold.[37] I have applied these latter results within the context of my study. Yet here, moreover, even assumption (a) has been relaxed: voters need not be assumed to share the common goal of the common good, but may have their own self-interest (or, indeed, any interest or cause whatsoever) as their goal — as long as their competence regarding this goal translates into required levels of (average) CJT-competence.[38] In addition, I have shown that this result holds for a novel democratic decision rule, whose (weak) collective optimality heretofore is largely unexplored.[39]

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[36] Ladha (1993: 69).

[37] There has also been some development concerning a fourth implicit assumption not mentioned by Ladha (1993): (d) voters vote sincerely (or non-strategically). This assumption has been challenged within a game-theoretic approach to voting, in which the individual voter's probability to vote correctly is not understood as her individual competence but rather as the other voters' confidence in the correctness of her vote. For one of the pioneering papers of this approach, see Austen-Smith and Banks (1996). Cf. even Dietrich (2008). It is an interesting further question — albeit one that goes beyond the limits of the present study — whether and how the case for the weighted majority rule could be restated within such a game-theoretic model.

[38] There have been other such efforts; see e.g. Miller (1986).

[39] As a corollary, since the weighted majority rule is extensionally equivalent to the simple majority rule in equal-stakes cases, my results apply to the latter as well.

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